

**UNIT III THREE PHASE INDUCTION MOTOR**

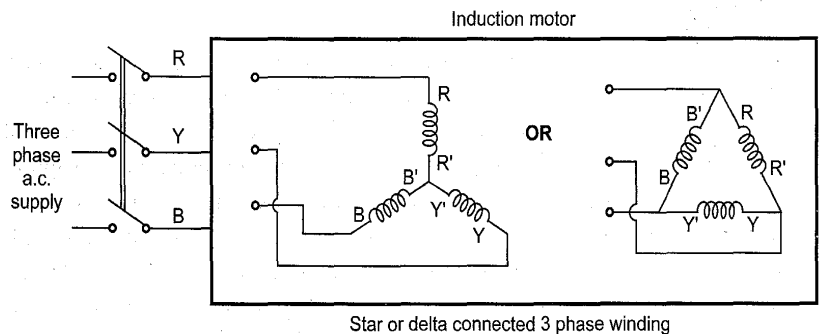
Constructional details – Types of rotors – Principle of operation – Slip – cogging and crawling- Equivalent circuit – Torque-Slip characteristics - Condition for maximum torque – Losses and efficiency – Load test - No load and blocked rotor tests - Circle diagram – Separation of losses – Double cage induction motors – Induction generators – Synchronous induction motor.

**1. Explain how a three phase supply produces a rotating magnetic field of constant value at constant speed with vector diagrams.**

The working principle of three phase induction motors is based on the production of rotating magnetic field. The rotating magnetic field can be defined as the field or flux having constant amplitude but whose axis is continuously rotating in a plane with a certain speed.

A three phase induction motor consists of three phase winding as its stationary part called stator. The three phase stator winding is connected in star or delta. The three phase windings are displaced from each other by 120°. The windings are supplied by a balanced three phase a.c. supply.

The three phase currents flow simultaneously through the windings and are displaced from each other by 120° electrical. Each alternating phase current produces its own flux which is sinusoidal. So all three fluxes are sinusoidal and are separated from each other by 120°. If the phase sequence of the windings is R-Y-B, then mathematical equations for the instantaneous values of the three fluxes  $\Phi_R$ ,  $\Phi_Y$  and  $\Phi_B$  can be written as,



$$\begin{aligned} \Phi_R &= \Phi_m \sin(\omega t) = \Phi_m \sin \theta & \text{(i)} \\ \Phi_Y &= \Phi_m \sin(\omega t - 120^\circ) = \Phi_m \sin(\theta - 120^\circ) & \text{(ii)} \\ \Phi_B &= \Phi_m \sin(\omega t - 240^\circ) = \Phi_m \sin(\theta - 240^\circ) & \text{(iii)} \end{aligned}$$

Let  $\Phi_R$ ,  $\Phi_Y$  and  $\Phi_B$  be the instantaneous values of three fluxes. The resultant flux is the phasor addition of all three.

Case 1:  $\theta = 0^\circ$

Substituting in equation (i), (ii) and (iii),

$$\begin{aligned} \Phi_R &= \Phi_m \sin \theta \\ \Phi_Y &= \Phi_m \sin(-120^\circ) = -0.866 \Phi_m \\ \Phi_B &= \Phi_m \sin(-240^\circ) = +0.866 \Phi_m \end{aligned}$$

BD is drawn perpendicular from B on  $\Phi_T$ . It bisects  $\Phi_T$ .

$$OD = DA = \Phi_T / 2$$

In triangle OBD,  $\angle BOD = 30^\circ$

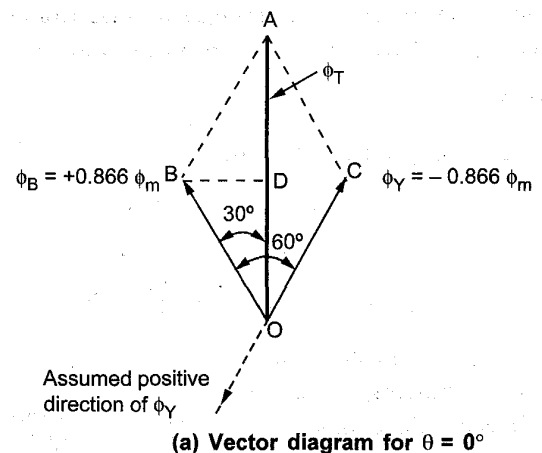
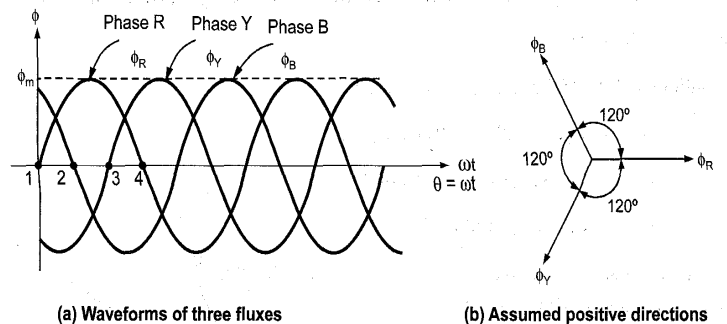
$$\cos 30^\circ = \frac{OD}{OB} = \frac{\Phi_T/2}{0.866 \Phi_m}$$

$$\Phi_T = 2 \times 0.866 \Phi_m \times \cos 30^\circ = 1.5 \Phi_m$$

So magnitude of  $\Phi_T$  is  $1.5 \Phi_m$  and its position is vertically upwards at  $\theta = 0^\circ$ .

Case 2:  $\theta = 60^\circ$

$$\Phi_R = \Phi_m \sin 60^\circ = 0.866 \Phi_m$$



$$\Phi_Y = \Phi_m \sin (-60^\circ) = -0.866 \Phi_m$$

$$\Phi_B = \Phi_m \sin (-180^\circ) = 0$$

Doing the same construction, drawing perpendicular from B on  $\Phi_T$  at D we get the same result as,

$$\Phi_T = 1.5 \Phi_m$$

But it can be seen that though its magnitude is  $1.5 \Phi_m$ , it has rotated through  $60^\circ$  in space, in clockwise direction, from its previous position.

Case 3:  $\theta = 120^\circ$

$$\Phi_R = \Phi_m \sin 120^\circ = 0.866 \Phi_m$$

$$\Phi_Y = \Phi_m \sin 0 = 0$$

$$\Phi_B = \Phi_m \sin (-120^\circ) = -0.866 \Phi_m$$

Doing the same construction, drawing perpendicular from B on  $\Phi_T$  at D we get the same result as,

$$\Phi_T = 1.5 \Phi_m$$

But it can be seen that though its magnitude is  $1.5 \Phi_m$ , it has rotated through  $60^\circ$  in space, in clockwise direction, from its previous position.

And from its position at  $\theta = 0^\circ$ , it has rotated through  $120^\circ$  in space, in clockwise direction.

Case 3:  $\theta = 180^\circ$

$$\Phi_R = \Phi_m \sin 180^\circ = 0$$

$$\Phi_Y = \Phi_m \sin (60^\circ) = 0.866 \Phi_m$$

$$\Phi_B = \Phi_m \sin (-60^\circ) = -0.866 \Phi_m$$

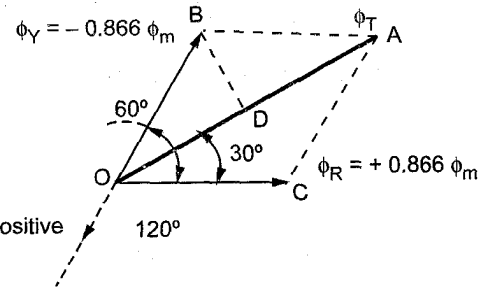
Doing the same construction, drawing perpendicular from B on  $\Phi_T$  at D we get the same result as,

$$\Phi_T = 1.5 \Phi_m$$

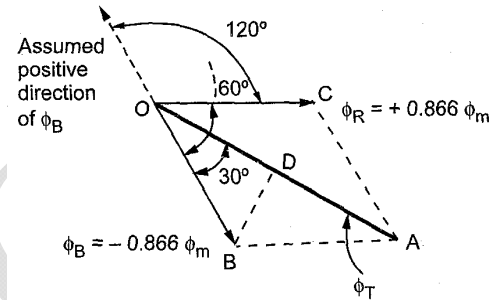
But it can be seen that though its magnitude is  $1.5 \Phi_m$ , it has rotated through  $60^\circ$  in space, in clockwise direction, from its previous position.

And from its position at  $\theta = 0^\circ$ , it has rotated through  $180^\circ$  in space, in clockwise direction.

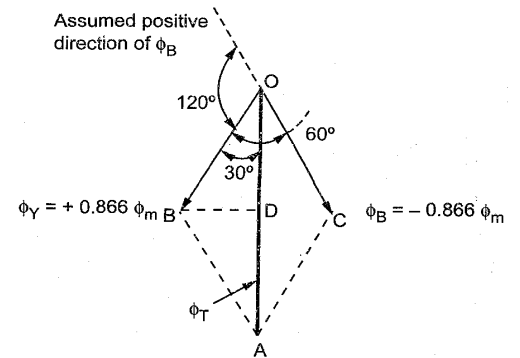
So for an electrical half cycle of  $180^\circ$ , the resultant  $\Phi_T$  has also rotated through  $180^\circ$ .



(b) Vector diagram for  $\theta = 60^\circ$



(c) Vector diagram for  $\theta = 120^\circ$



(d) Vector diagram for  $\theta = 180^\circ$

**2. Describe the constructional features of squirrel cage and slip ring induction motors. Discuss the merits of one over other.**

The induction motor consists of two main parts,

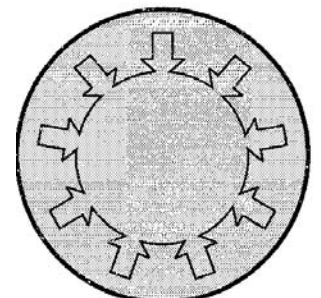
1. The part i.e. three phase windings, which is stationary called stator.
2. The part which rotates and is connected to the mechanical load through shaft called rotor.

The conversion of electrical power to mechanical power takes place in a rotor.

Hence rotor develops a driving torque and rotates.

**Stator**

- The stator has a laminated type of construction made up of stampings which are 0.4 to 0.5 mm thick.
- The stampings are slotted on its periphery to carry the stator winding.
- The stampings are insulated from each other to keep the iron losses to a minimum value.
- The number of stampings are stamped together to build the stator core.



Stator Lamination

- The built up core is then fitted in a casted or fabricated steel frame.
- The choice of material for the stampings is generally silicon steel, which minimizes the hysteresis loss.
- The slots on the periphery of the stator core carries a three phase winding, connected either in star or delta.
- This three phase winding is called stator winding. It is wound for definite number of poles.
- This winding when excited by a three phase supply produces a rotating magnetic field.
- The choice of number of poles depends on the speed of the rotating magnetic field required.
- The radial ducts are provided for the cooling purpose.

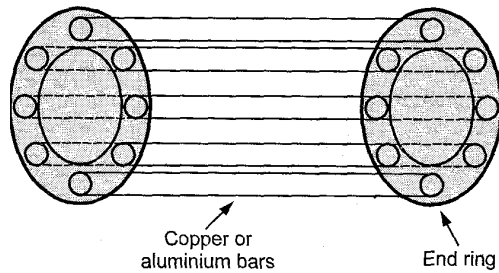
**Rotor**

The two types of rotor constructions used for induction motors are,

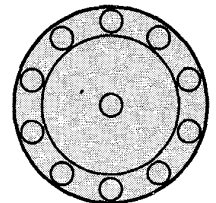
1. Squirrel cage rotor and 2. Slip ring or wound rotor

**Squirrel Cage Rotor**

- The rotor core is cylindrical and slotted on its periphery.
- The rotor consists of uninsulated copper or aluminium bars called rotor conductors.
- The bars are placed in the slots. These bars are permanently shorted at each end with the help of conducting copper ring called end ring.
- The bars are usually brazed to the end rings to provide good mechanical strength.
- The entire structure looks like a cage, forming a closed electrical circuit. So the rotor is called squirrel cage rotor.
- As the bars are permanently shorted to each other through end ring, the entire rotor resistance is very very small. Hence this rotor also called short circuited rotor.
- As rotor itself is short circuited, no external resistance can have any effect on the rotor resistance. Hence no external resistance can be introduced in the rotor circuit. So slip ring and brush assembly is not required for this rotor. Hence the construction of this rotor is very simple.



(a) Cage type structure of rotor

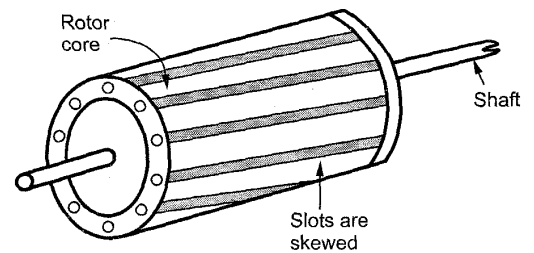


(b) Symbolic representation

In this type of rotor, the slots are not arranged parallel to the shaft axis but are skewed as shown in the Fig..

The advantages of skewing are,

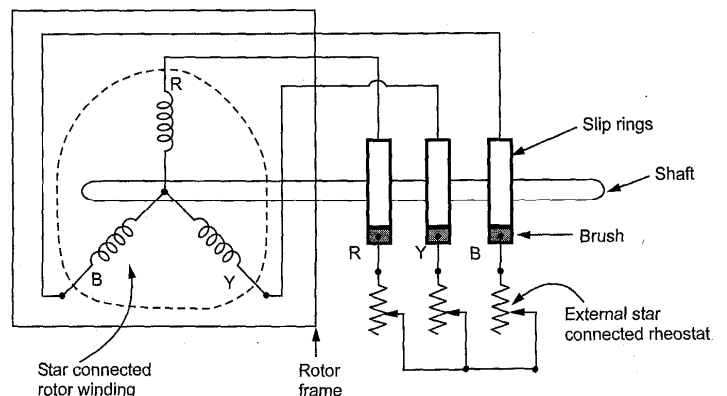
1. A magnetic hum i.e. noise gets reduced due to skewing hence skewing makes the motor operation quieter.
2. It makes the motor operation smooth.
3. The stator and rotor teeth may get magnetically locked. Such a tendency of magnetic locking gets reduced due to skewing.
4. It increases the effective transformation ratio between stator and rotor.



Skewing in rotor construction

**Slip Ring Rotor or Wound Rotor**

- In this type of construction, rotor winding is exactly similar to the stator.
- The rotor carries a three phase star or delta connected, distributed winding, wound for same number of poles as that of stator.
- The rotor construction is laminated and slotted.
- The slots contain the rotor winding. The three ends of three phase winding, available after connecting the winding in star or delta, are permanently connected to the slip rings.
- The slip rings are mounted on the same shaft.



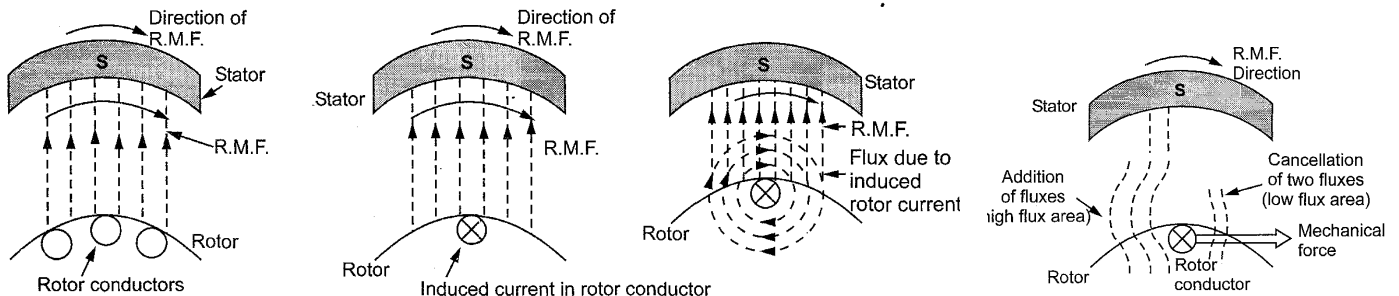
- In this type of rotor, the external resistances can be added with the help of brushes and slip ring arrangement, in series with each phase of the rotor winding.

**3. Compare squirrel cage and wound rotor constructions of induction motor.**

S.No	Wound or Slip ring rotor	Squirrel cage rotor
1.	Rotor consists of a three phase winding similar to the stator winding.	Rotor consists of bars, which are shorted at the ends with the help of end rings.
2.	Construction is complicated.	Construction is simple
3.	Resistance can be added externally:	As permanently shorted, external resistance cannot be added
4.	Slip rings and brushes are present to add external resistance.	Slip rings and brushes are absent
5.	Frequent maintenance is necessary	Maintenance free
6.	Rotors are costly	Rotors are cheap
7.	High starting torque can be obtained	Moderate starting torque
8.	Rotor must be wound for same number of poles	The rotor automatically adjusts itself for the same number of poles as that of stator
9.	Rotor resistance starter can be used	Rotor resistance starter cannot be used
10.	Rotor copper losses are high hence efficiency is less	Rotor copper losses are less hence efficiency is high
11.	Speed control by rotor resistance is possible	Speed control by rotor resistance is not possible
12.	Used for lifts, hoists, cranes, elevators, compressors etc.	Used for lathes, drilling machines, fans, blowers, waterpumps, grinders, printing machines etc.

**4. Describe the principle of operation of a 3 phase induction motor with a neat sketch. Explain why a rotor is forced to rotate in the direction of rotating magnetic field.**

When a three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced. The speed of this rotating magnetic field is synchronous speed,  $N_s$  r.p.m. The R.M.F. gets cut by rotor conductors as R.M.F. sweeps over rotor conductors. Whenever conductor cuts the flux, e.m.f. gets induced in it according to Faraday’s Law of electro-magnetic induction. As rotor forms closed circuit, induced e.m.f. circulates current through rotor called rotor current whose direction is given by Lenz’s law. According to Lenz’s law the direction of induced current in the rotor is so as oppose the cause producing it. The cause of rotor current is the induced e.m.f. which is induced because of relative motion present between the rotating magnetic field and the rotor conductors. Hence to oppose the relative motion i.e. to reduce the relative speed, the rotor experiences a torque in the same direction as that of R.M.F. and tries to catch up the speed of rotating magnetic field.



So,

$N_s$  = Speed of rotating magnetic field in r.p.m.

$N$  = Speed of rotor i.e. motor in r.p.m.

$N_s - N$  = Relative speed between the two, rotating magnetic field and the rotor conductors.

Thus rotor always rotates in same direction as that of R.M.F.

**5. Define the term slip of an induction motor. Explain its significance.**

Slip of the induction motor is defined as the difference between the synchronous speed ( $N_s$ ) and actual speed of rotor i.e. motor ( $N$ ) expressed as a fraction of the synchronous speed ( $N_s$ ). This is also called absolute slip or fractional slip and is denoted as  $s$ .

$$s = \frac{N_s - N}{N_s}$$

The percentage slip is expressed as,

$$\% s = \frac{N_s - N}{N_s} \times 100$$

In terms of slip, the actual speed of motor ( $N$ ) can be expressed as,

$$N = N_s (1 - s)$$

At start, motor is at rest and hence its speed  $N$  is zero.

$$s = 1 \text{ at start}$$

This is maximum value of slip  $s$  possible for induction motor which occurs at start. While  $s = 0$  gives us  $N = N_s$  which is not possible for an induction motor. So slip of induction motor cannot be zero under any circumstances.

Practically motor operates in the slip range of 0.01 to 0.05 i.e. 1 % to 5 %. The slip corresponding to full load speed of the motor is called full load slip.

**6. Explain the effect of slip on the rotor parameters****(a) Effect on Rotor Frequency**

The speed of rotating magnetic field is,

$$N_s = 120f/P \quad (i)$$

where  $f$  = frequency of supply in Hz.

At start when  $N = 0$ ,  $s = 1$  and stationary rotor has maximum relative motion with respect to R.M.F. Hence maximum e.m.f. gets induced in the rotor at start. The frequency of this induced e.m.f. at start is same as that of supply frequency.

In running condition the magnitude of the induced emf decreases as its frequency.

$$(N_s - N) = 120f_r / P \quad (ii)$$

Where,  $f_r$  = frequency of rotor induced emf in running condition at slip speed ( $N_s - N$ )

Dividing equation (ii) by (i), we get

$$\frac{N_s - N}{N_s} = \frac{120 f_r / P}{120 f / P}$$

Therefore  $s = f_r / f$  or  $f_r = sf$

Thus frequency of rotor induced e.m.f. in running condition ( $f_r$ ) is slip times the supply frequency ( $f$ ).

**(b) Effect on Magnitude of Rotor Induced E.M.F.**

$E_2$  = Rotor induced e.m.f. per phase on standstill condition

As rotor gains speed, the relative speed between rotor and rotating magnetic field decreases and hence induced e.m.f. in rotor also decreases as it is proportional to the relative speed  $N_s - N$ . Let

$E_{2r}$  = Rotor induced e.m.f. per phase in running condition

$$E_2 \propto N_s \quad \text{and} \quad E_{2r} \propto (N_s - N)$$

$$\frac{E_{2r}}{E_2} = \frac{N_s - N}{N_s} \quad \text{but} \quad \frac{N_s - N}{N_s} = \text{slip } s$$

$$\therefore \frac{E_{2r}}{E_2} = s$$

$$E_{2r} = sE_2$$

The magnitude of the induced e.m.f. in the rotor also reduces by slip times the magnitude of induced e.m.f. at standstill condition.

**(c) Effect on Rotor Resistance and Reactance**

$R_2$  = Rotor resistance per phase on standstill

$X_2$  = Rotor reactance per phase on standstill

Now at standstill,  $f_r = f$  hence if  $L_2$  is the inductance of rotor per phase,

$$X_2 = 2\pi f_r L_2 = 2\pi f L_2$$

While  $R_2 =$  Rotor resistance in  $\Omega$  /ph

Now in running condition,  $f_r = sf$  hence

$$X_{2r} = 2\pi f_r L_2 = 2\pi sf L_2 = s \cdot 2\pi f L_2$$

$$X_{2r} = s X_2$$

Where  $X_{2r} =$  Rotor reactance in running condition

If  $Z_2 =$  Rotor impedance on standstill condition  
 $= R_2 + j X_2 \Omega$  / ph

$$Z_2 = \sqrt{R_2^2 + X_2^2} \Omega$$
 / ph

While  $Z_{2r} =$  rotor impedance in running condition  
 $= \sqrt{R_2^2 + (sX_2)^2} \Omega$  / ph

**(d) Effect on Rotor Power Factor**

$\cos \phi_2 =$  rotor power factor on standstill

$$= \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$\cos \phi_{2r} =$  rotor power factor in running condition

$$= \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

**(e) Effect on Rotor Current**

$I_2 =$  Rotor current per phase on standstill condition

$$= \frac{E_2 \text{ per phase}}{Z_2 \text{ per phase}} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \text{ A}$$

$I_{2r} =$  Rotor current per phase in running condition.

$$= \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \text{ A}$$

**7. Derive the torque equation for a three phase induction motor.**

The torque produced in the induction motor depends on the following factors.

1. The part of rotating magnetic field which reacts with rotor and is responsible to produce induced e.m.f. in rotor.
2. The magnitude of rotor current in running condition.
3. The power factor of the rotor circuit in running condition.

$$T \propto \Phi I_{2r} \cos \phi_{2r} \quad (1)$$

Where  $\Phi =$  Flux responsible to produce induced e.m.f.

$I_{2r} =$  Rotor running current

$\cos \phi_{2r} =$  Running p.f. of rotor

The flux  $\Phi$  produced by stator is proportional to  $E_1$  i.e. stator voltage.

$$\Phi \propto E_1 \quad (2)$$

While  $E_1$  and  $E_2$  are related to each other through ratio of stator turns to rotor turns i.e.  $K$ .

$$\frac{E_1}{E_2} = K \quad (3)$$

Using equation (3) in (2),

$$E_2 \propto \Phi \quad (4)$$

Thus in equation (1),  $\Phi$  can be replaced by  $E_2$

$$\text{While } I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \text{ A}$$

$$\text{And } \cos \phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Therefore, } T \propto E_2 \cdot \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}$$

$$T = \frac{k s E_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}$$

where  $k = \text{constant of proportionality}$

$$= \frac{3}{2\pi n_s} \quad [n_s = \text{synchronous speed in r.p.s} = N/60]$$

$$T = \frac{3}{2\pi n_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}$$

### Starting torque

Starting torque is nothing but the torque produced by an induction motor at start. At start,  $N = 0$  and slip  $s = 1$ . So putting  $s = 1$  in the torque equation, the starting torque,

$$T_{st} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2 R_2}{(R_2^2 + X_2^2)} \text{ N-m}$$

### 8. Derive the condition for the maximum torque in a three phase induction motors. Also obtain the expression for the maximum torque.

For maximum torque,

$$\frac{dT}{ds} = 0$$

$$\text{Where } T = \frac{k s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

As load on motor changes, its speed changes and hence slip changes. This slip decides the torque produced corresponding to the load demand.

$$\frac{dT}{ds} = \frac{d}{ds} \left( \frac{k s E_2^2 R_2}{R_2^2 + (sX_2)^2} \right)$$

$$\therefore \frac{dT}{ds} = \frac{(k s E_2^2 R_2) \frac{d}{ds} (R_2^2 + s^2 X_2^2) - (R_2^2 + s^2 X_2^2) \frac{d}{ds} (k s E_2^2 R_2)}{(R_2^2 + s^2 X_2^2)^2} = 0$$

$$\therefore k s E_2^2 R_2 [2s X_2^2] - (R_2^2 + s^2 X_2^2) (k E_2^2 R_2) = 0$$

$$\therefore 2 s^2 k X_2^2 E_2^2 R_2 - R_2^2 k E_2^2 R_2 - k s^2 X_2^2 E_2^2 R_2 = 0$$

$$\therefore k s^2 X_2^2 E_2^2 R_2 - R_2^2 k E_2^2 R_2 = 0$$

$$\therefore s^2 X_2^2 - R_2^2 = 0 \quad \text{Taking } k E_2^2 R_2 \text{ common.}$$

$$s^2 = \frac{R_2^2}{X_2^2} \quad \text{or } s = \frac{R_2}{X_2}$$

This is the slip at which the torque is maximum and is denoted by  $s_m$ .

$$s_m = \frac{R_2}{X_2}$$

By substituting  $s_m = \frac{R_2}{X_2}$  in the torque equation, maximum torque  $T_m$  is

$$T_m = \frac{k s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2} = \frac{k E_2^2}{2 X_2} \text{ N-m}$$

From the expression of  $T_m$ , it can be observed that

1. It is inversely proportional to the rotor reactance.
2. It is directly proportional to the square of the rotor induced e.m.f. at standstill.
3. The maximum torque is not dependent on the rotor resistance  $R_2$ . But the slip at which it occurs i.e. speed at which it occurs depends on the value of rotor resistance  $R_2$ .

**9. Draw and explain a typical torque-speed characteristic for a 3 phase induction motor. Explain the relation between torque and slip before and after the maximum torque. Show the stable region in the graph.**

The curve obtained by plotting torque against slip from  $s = 1$  (at start) to  $s = 0$  (at synchronous speed) is called torque-slip characteristics of the induction motor.

$$T \propto \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N-m}$$

$E_2$  is also constant. Therefore,

$$T \propto \frac{sR_2}{R_2^2 + (sX_2)^2} \text{ N-m}$$

**i) Low slip region**

In low slip region, 's' is very very small. Due to this, the term  $(sX_2)^2$  is so small as compared to  $R_2^2$  that it can be neglected.

Therefore  $T = \frac{sR_2^2}{R_2^2} \propto s$  as  $R_2$  is constant.

Hence in low slip region torque is directly proportional to slip. So as load increases, speed decreases, increasing the slip. This increases the torque which satisfies the load demand.

Hence the graph is straight line in nature.

At  $N = N_s$ ,  $s = 0$  hence  $T = 0$ . As no torque is generated at  $N = N_s$ , motor stops if it tries to achieve the synchronous speed. Torque increases linearly in this region, of low slip values.

**ii) High slip region**

In this region, slip is high i.e. slip value is approaching to 1. Here it can be assumed that the term  $R_2^2$  is very very small as compared to  $(sX_2)^2$ . Hence neglecting  $R_2^2$  from the denominator, we get

$$T \propto \frac{sR_2}{(sX_2)^2} \propto \frac{1}{s}$$

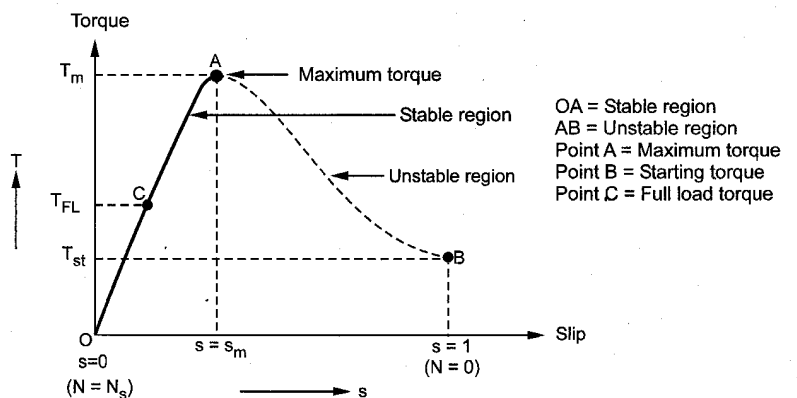
So in high slip region torque is inversely proportional to the slip. Hence its nature is like rectangular hyperbola.

Now when load increases, load demand increases but speed decreases. As speed decreases, slip increases. In high slip region as  $T \propto 1/s$ , torque decreases as slip increases. But torque must increase to satisfy the load demand. As torque decreases, due to extra loading effect, speed further decreases and slip further increases. Hence speed further drops. Eventually motor comes to standstill condition. The motor cannot continue to rotate at any point in this high slip region. Hence this region is called unstable region of operation.

So torque - slip characteristics has two parts,

1. Straight line called stable region of operation.
2. Rectangular hyperbola called unstable region of operation.

In low slip region, as load increases, slip increases and torque also increases linearly. The maximum torque, the motor can produce as load increases is  $T_m$  which occurs at  $s = s_m$ . So linear behavior continues till  $s = s_m$ .





If load is increased beyond this limit, motor slip acts dominantly pushing motor into high slip region. Due to unstable conditions, motor comes to standstill condition at such a load. Hence  $T_m$  i.e. maximum torque which motor can produce is also called **breakdown torque or pull out torque**. So range  $s = 0$  to  $s = s_m$  is called low slip region, known as stable region of operation. Motor always operates at a point in this region. And range  $s = s_m$  to  $s = 1$  is called high slip region which is rectangular hyperbola, called unstable region of operation. Motor cannot continue to rotate at any point in this region.

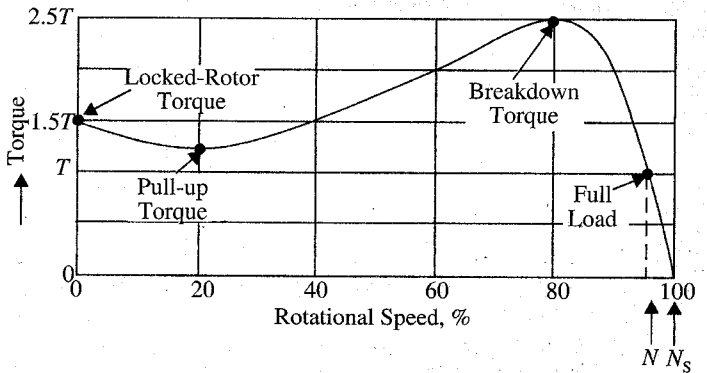
At  $s = 1, N = 0$  i.e. at start, motor produces a torque called starting torque denoted as  $T_{st}$ .

**10. Explain the Torque/speed Curve and discuss briefly the effect on the speed-torque characteristics of an induction motor.**

The torque developed by a 3-phase motor depends on its speed but the relation between the two cannot be represented by a simple equation. It is easier to show the relationship in the form of a curve.

In this diagram,  $T$  represents the nominal full load torque of the motor. The starting torque (at  $N = 0$ ) is  $1.5 T$  and the maximum torque (break down torque) is  $2.5 T$ .

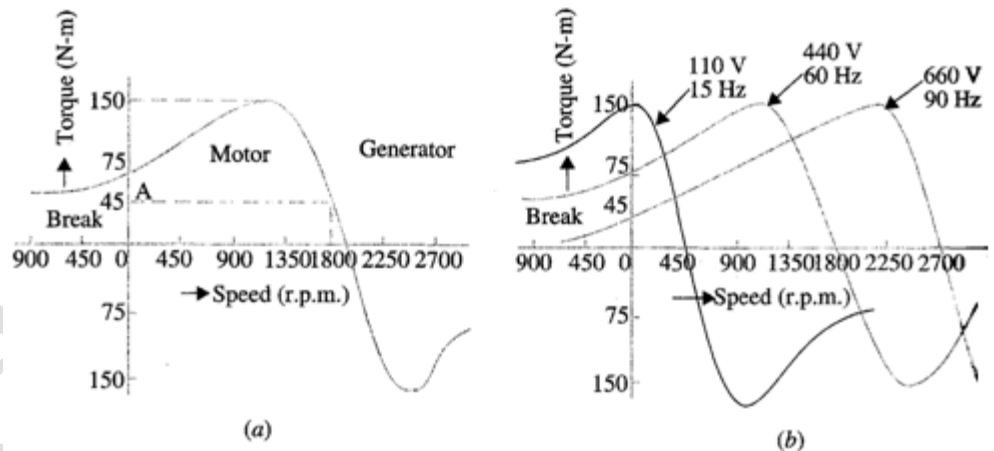
At full-load, the motor runs at a speed of  $N$ . When mechanical load increases, the motor speed decreases till the motor torque again becomes equal to the load torque. As long as the two torques are in balance, the motor will run at constant (but lower) speed. However If the load torque exceeds  $2.5 T$ , the motor will suddenly stop.



**Shape of Torque/speed curve:**

For a squirrel-cage induction motor (SCIM) shape of its torque/speed curve depends on the voltage and frequency applied to its stator. If  $f$  is fixed,  $T \propto V^2$ .

Also synchronous speed depends on the supply frequency. In practice, supply voltage and frequency are varied in the same proportion in order to maintain a constant flux in the air-gap. For example, if voltage is doubled, then frequency is also doubled. Under these conditions, shape of the torque/speed curve remains the same but its position along the X-axis (speed axis) shifts with frequency. Since the shape of the torque/speed curve remains the same at all frequencies, it follows that torque developed by a SCIM is the same whenever slip speed is the same.



**11. Prove that to increase the starting torque an extra resistance must be added in the rotor circuit.**

In slip ring induction motor, externally resistance can be added in the rotor.

$R_2$  = rotor resistance per phase

$$\text{Torque } T \propto \frac{sE^2R_2}{R_2^2 + (sX)^2}$$

Externally resistance is added in each phase of rotor through slip rings

$R_2'$  = new rotor resistance per phase

$$\text{Torque } T \propto \frac{sE^2R_2'}{R_2'^2 + (sX_2)^2}$$

The starting torque at  $s = 1$   $R_2$  and  $R_2'$  are

$$T_{st} \propto \frac{E^2R_2}{R_2^2 + X_2^2}$$

and

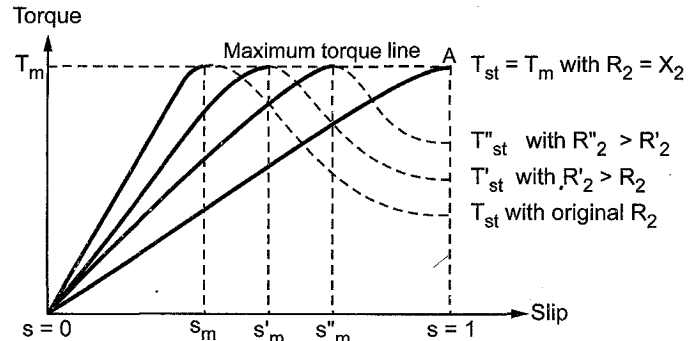
$$T_{st}' \propto \frac{E^2R_2'}{R_2'^2 + X_2^2}$$

Maximum torque  $T_m \propto \frac{E^2}{2X_2}$

For  $R_2$ ,  $s_m = \frac{R_2}{X_2}$  where  $T_m$  occurs

For  $R_2'$ ,  $s_m' = \frac{R_2'}{X_2}$  where same  $T_m$  occurs.

As  $R_2' > R_2$ , the slip  $s_m' > s_m$ . the starting torque  $T_{st}'$  for  $R_2'$  is more than  $T_{st}$  for  $R_2$ . Thus by controlling the rotor resistance the starting torque can be controlled. If resistance is further added to rotor, maximum torque is constant but slip at which it occurs increases and so starting torque increases.



**12. Discuss the different power stages of a 3 phase induction motor with losses with the help of a power flow diagram.**

Induction motor converts an electrical power supplied to it into mechanical power. The various stages in this conversion is called power flow in an induction motor.

The three phase supply given to the stator is the net electrical input to the motor.

The net input electrical power supplied to the motor is,

$$P_{in} = \sqrt{3} V_L I_L \cos \phi$$

This is the stator input.

The part of this power is utilized to supply the losses in the stator which are stator core as well as copper losses.

The remaining power is delivered to the rotor magnetically through the air gap with the help of rotating magnetic field. This is called rotor input denoted as  $P_2$ .

$$P_2 = P_{in} - \text{Stator losses (core + copper)}$$

The rotor is not able to convert its entire input to the mechanical as it has to supply rotor losses. The rotor losses are dominantly copper losses as rotor iron losses are very small and hence generally neglected. So rotor losses are rotor copper losses denoted as

$$P_c = 3 I_{2r}^2 R_2$$

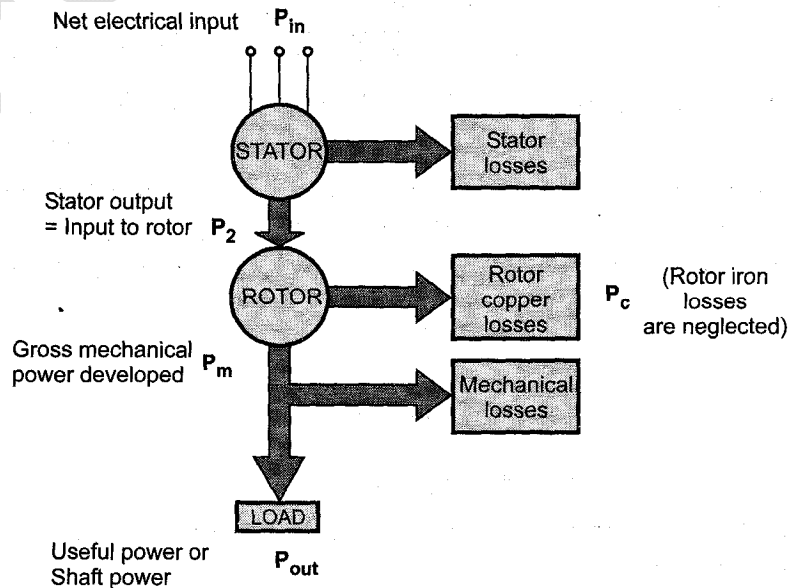
Where  $I_{2r}$  = Rotor current per phase in running condition

$$R_2 = \text{Rotor resistance per phase}$$

After supplying these losses, the remaining part of  $P_2$  is converted into mechanical which is called gross mechanical power developed by the motor denoted as  $P_m$ .

$$P_m = P_2 - P_c$$

Part of  $P_m$  is utilized to provide mechanical friction and windage. Finally the power is available to the load at the shaft. This is called net output of the motor denoted as  $P_{out}$ . This is also called shaft power.



$$\text{Rotor efficiency} = \frac{\text{rotor output}}{\text{rotor input}} = \frac{\text{gross mechanical power developed}}{\text{rotor input}} = \frac{P_m}{P_2}$$

$$\text{Net motor efficiency} = \frac{\text{Net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_{in}}$$

**13. Derive the relation between input, rotor copper losses and mechanical power developed in terms of a slip of a three phase induction motor.**

Let T = gross torque developed by motor in N-m

$$\text{Power } P = T \times \omega$$

Where  $\omega$  = angular speed =  $\frac{2\pi N}{60}$ , N = speed in r.p.m

Input to the rotor  $P_2$  is from stator side through rotating magnetic field which is at synchronous speed  $N_s$ .

$$P_2 = T \times \omega_s, \text{ where } \omega_s = \frac{2\pi N_s}{60} \text{ rad/sec} \quad (1)$$

$$P_2 = T \times \frac{2\pi N_s}{60} \text{ where } N_s \text{ is in r.p.m}$$

Rotor output is gross mechanical power developed  $P_m$ . rotor gives the output at speed n.

$$P_m = T \times \omega \text{ where } \omega = \frac{2\pi N}{60}$$

$$P_m = T \times \frac{2\pi N}{60} \quad (2)$$

$$\text{Rotor copper loss } P_c = P_2 - P_m = T \times \frac{2\pi N_s}{60} - T \times \frac{2\pi N}{60} = T \times \frac{2\pi}{60} (N_s - N) \quad (3)$$

Dividing equation (3) by (1),

$$\frac{P_c}{P_2} = \frac{T \times \frac{2\pi}{60} (N_s - N)}{T \times \frac{2\pi N_s}{60}} = \frac{N_s - N}{N_s} = s$$

Rotor copper loss  $P_c = s \times$  rotor input  $P_{in}$

Thus total rotor copper loss is slip times the rotor input.

$$P_2 - P_c = P_m$$

$$P_2 - sP_2 = P_m$$

$$(1-s) P_2 = P_m$$

Thus gross mechanical power developed is  $(1 - s)$  times the rotor input.

The relationship can be expressed in the ratio form as,

$$P_2 : P_c : P_m :: 1 : s : (1-s)$$

**14. Develop an equivalent circuit of a 3 phase induction motor. What do the various parameters represent? Represent the approximate equivalent circuit and state its significance.**

The energy transfer from stator to rotor of the induction motor takes place entirely with the help of a flux mutually linking the two. Thus stator acts as a primary while the rotor acts as a rotating secondary when induction motor is treated as transformer.

If

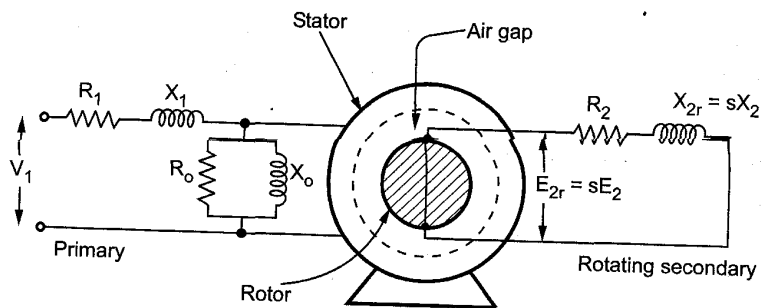
$E_1$  = Induced voltage in stator per phase

$E_2$  = Rotor induced e.m.f. per phase on standstill

$$K = \frac{\text{rotor turns}}{\text{stator turns}} = \frac{E_2}{E_1}$$

$E_{2r}$  = Rotor induced e.m.f. in running condition per phase

$R_2$  = Rotor resistance per phase



$X_{2r}$  = Rotor reactance per phase in running condition

$R_1$  = Stator resistance per phase

$X_1$  = Stator reactance per phase

When induction motor is on no load, it draws a current from the supply to produce the flux in air gap and to supply iron losses.

This current  $I_0$  has two components.

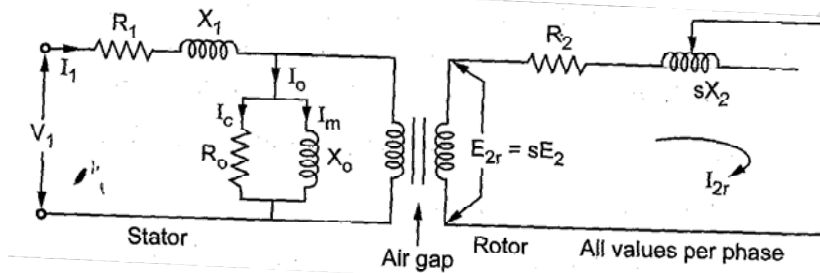
1.  $I_c$  = Active component which supplies no load losses
2.  $I_m$  = Magnetising component which sets up flux in core and air gap

These two currents give us the elements of an exciting branch as,

$$R_0 = \text{Representing no load losses} = \frac{V_1}{I_c}$$

$$X_0 = \text{representing flux set up} = \frac{V_1}{I_m}$$

The equivalent circuit of induction motor is given by,



$$\begin{aligned} I_{2r} &= \text{Rotor current per phase in running condition.} \\ &= \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \text{ A} \\ &= \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}} \end{aligned}$$

So it can be assumed that equivalent rotor circuit in the running condition has fixed reactance  $X_2$ , fixed voltage  $E_2$  but a variable resistance  $R_2/s$ .

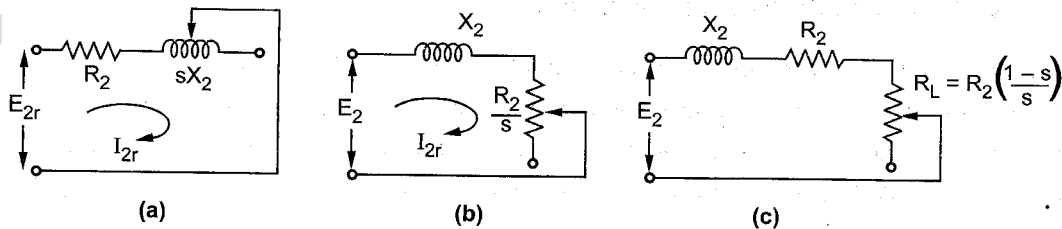
$$\frac{R_2}{s} = R_2 + \frac{R_2}{s} + R_2$$

$$\frac{R_2}{s} = R_2 + R_2 \left( \frac{1}{s} - 1 \right) = R_2 + R_2 (1-s) / s$$

So the variable rotor resistance  $R_2/s$  has two parts,

1. Rotor resistance  $R_2$  itself which represents copper loss.
2.  $R_2 (1 - s)/s$  which represents load resistance  $R_L$ . So it is electrical equivalent of mechanical load on the motor.

The rotor equivalent circuit is



Equivalent circuit referred to stator:

$$K = \frac{E_2}{E_1} = \text{transformation ratio}$$

$$E_2' = \frac{E_2}{K}$$

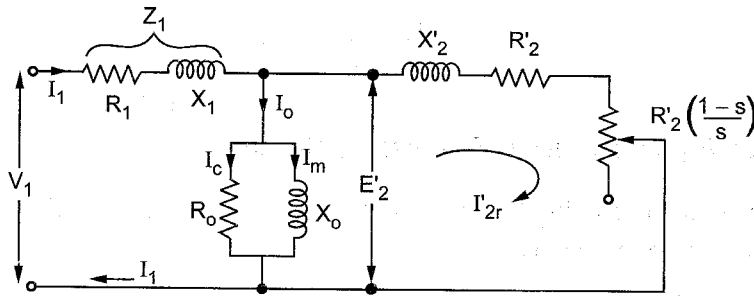
$$\text{Rotor current referred to stator } I_{2r}' = K I_{2r} = \frac{KsE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor reactance referred to stator } X_2' = \frac{X_2}{K^2}$$

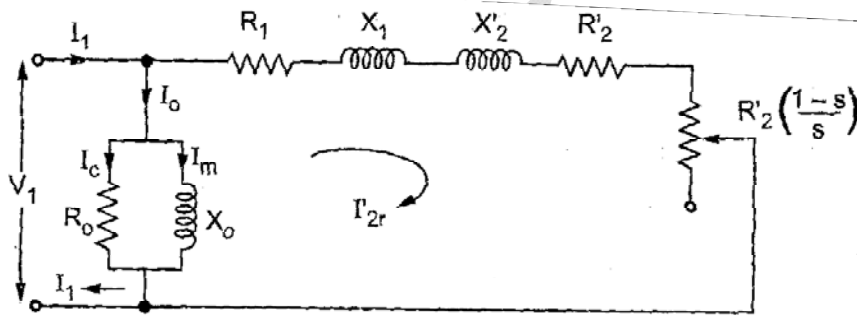
$$\text{Rotor resistance referred to stator } R_2' = \frac{R_2}{K^2}$$

$$\text{Mechanical load referred to stator } R_L' = \frac{R_L}{K^2} = \frac{R_2}{K^2} \frac{1-s}{s} = R_2' \frac{1-s}{s}$$

Equivalent circuit referred to stator is

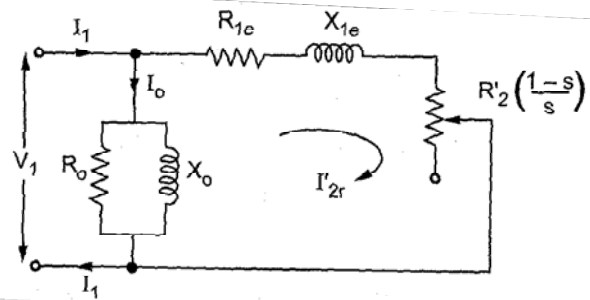


Approximate Equivalent circuit



$R_{1e}$  = equivalent resistance referred to stator =  $R_1 + R_2'$   
 $X_{1e}$  = Equivalent reactance referred to stator =  $X_1 + X_2'$

$$R_{1e} = R_1 + \frac{R_2}{K^2} \text{ and } X_{1e} = X_1 + \frac{X_2}{K^2}$$



**15. Derive the power equations from the equivalent circuit.**

$P_{in} = \text{input power} = 3 V_1 I_1 \cos \phi$

Where  $V_1 = \text{stator voltage per phase}$   
 $I_1 = \text{current drawn by stator per phase}$   
 $\cos \phi = \text{power factor of stator}$

stator core loss =  $I_m^2 R_o$

stator copper loss =  $3 I_1^2 R_1$

where  $R_1 = \text{stator resistance per phase}$

$P_2 = \text{rotor input} = \frac{3 I'_{2r}{}^2 R'_2}{s}$

$P_c = \text{rotor copper loss} = 3 I'_{2r}{}^2 R'_2$

Thus  $P_c = s P_2$

Gross mechanical power developed  $P_m = P_2 - P_c = \frac{3 I'_{2r}{}^2 R'_2}{s} - 3 I'_{2r}{}^2 R'_2 = 3 I'_{2r}{}^2 R'_2 (1-s) / s$

Torque developed  $T = \frac{P_m}{\omega} = \frac{3 I'_{2r}{}^2 R'_2 (1-s) / s}{\frac{2\pi N}{60}}$

Where  $N = \text{speed of motor}$

$N = N_s (1-s)$

Therefore  $T = \frac{3 I'_{2r}{}^2 R'_2 / s}{\frac{2\pi N_s}{60}} = 9.55 \times \frac{3 I'_{2r}{}^2 R'_2 / s}{N_s} \text{ N-m}$

$I'_{2r} = \frac{V_1}{(R_{1e} + R'_L) + j X_{1e}}$  where  $R'_L = R'_2 \frac{1-s}{s}$

$I'_{2r} = \frac{V_1}{\sqrt{(R_{1e} + R'_L)^2 + (X_{1e})^2}}$

**Maximum Power Output**

In this circuit, the exciting current  $I_0$  is neglected. Therefore

$I_1 = I'_{2r}$

Therefore  $I_1 = \frac{V_1}{\sqrt{(R_{1e} + R'_L)^2 + (X_{1e})^2}}$

Power supplied to the load

$P_{out} = 3 I_1^2 R'_L$   
 $= 3 \frac{V_1^2}{(R_{1e} + R'_L)^2 + (X_{1e})^2} R'_L$

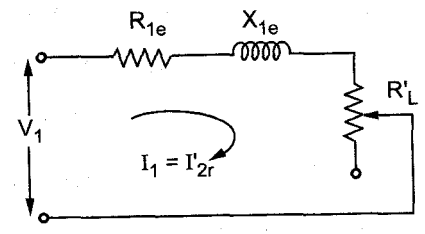
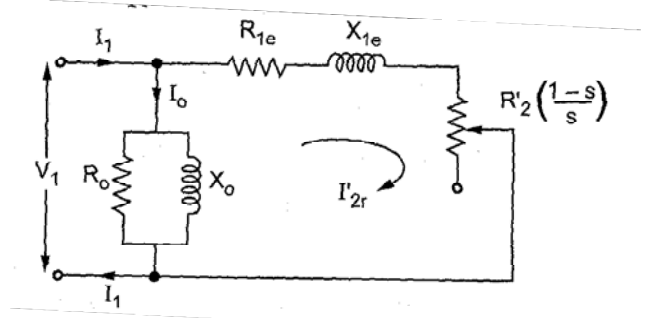
To obtain maximum output power,  $\frac{d P_{out}}{d R'_L} = 0$

$\therefore \frac{d}{d R'_L} \left[ \frac{3 V_1^2 (R'_L)}{[(R_{1e} + R'_L)^2 + (X_{1e})^2]} \right] = 0$

$\therefore [(R_{1e} + R'_L)^2 + (X_{1e})^2] [3 \cdot V_1^2] - 3 V_1^2 (R'_L) [2 (R_{1e} + R'_L)] = 0$

$\therefore (R_{1e} + R'_L)^2 + (X_{1e})^2 - 2 (R'_L) (R_{1e} + R'_L) = 0$  ... Taking  $3 V_1^2$  common

$\therefore R_{1e}^2 + (R'_L)^2 + 2 R_{1e} R'_L + X_{1e}^2 - 2 R_{1e} R'_L - 2(R'_L)^2 = 0$



$$\therefore R_{1e}^2 + X_{1e}^2 = (R'_L)^2$$

But  $Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} = \text{leakage impedance referred to stator}$

$$\therefore Z_{1e}^2 = (R'_L)^2$$

Therefore  $R'_L = Z_{1e}$

Hence the power output is maximum when the equivalent load resistance is equal to the standstill leakage impedance of the motor.

**Corresponding Slip**

$$R'_L = Z_{1e} = R_2 \frac{1-s}{s}$$

By solving we get

$$\text{Slip at maximum output, } s = \frac{R'_2}{R'_2 + Z_{1e}}$$

$$\text{Corresponding maximum gross power output } (P_{out})_{max} = 3 \frac{V_1^2}{(R_{1e} + Z_{1e})^2 + (X_{1e})^2} Z_{1e}$$

By solving

$$(P_{out})_{max} = \frac{3V_1^2}{2(R_{1e} + Z_{1e})} \text{ watts}$$

**16. Explain briefly about crawling and cogging.**

**CRAWLING**

Squirrel cage induction motors exhibit a tendency to run at very slow speeds (as low as one-seventh of their synchronous speed). This phenomenon is called as **crawling of an induction motor**.

This action is due to the fact that, flux wave produced by a stator winding is not purely sine wave. Instead, it is a complex wave consisting a fundamental wave and odd harmonics like 3rd, 5th, 7th etc. The fundamental wave revolves synchronously at synchronous speed  $N_s$ , whereas 3rd, 5th, 7th harmonics may rotate in forward or backward direction at  $N_s/3, N_s/5, N_s/7$  speeds respectively.

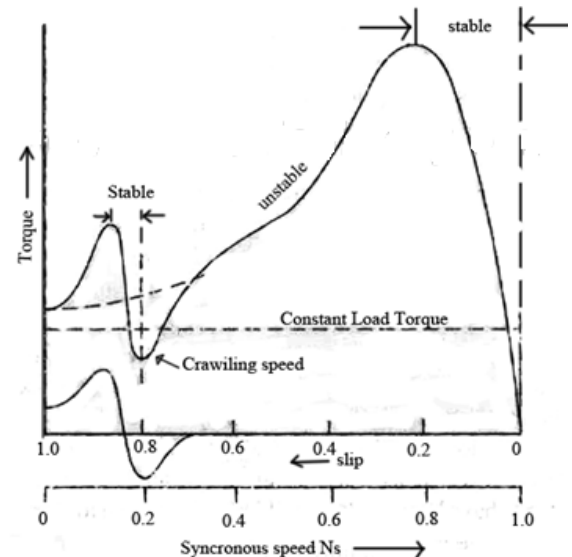
Hence, harmonic torques are also developed in addition with fundamental torque. 3rd harmonics are absent in a balanced 3-phase system. Hence 3rd harmonics do not produce rotating field and torque.

The total motor torque now consist three components as:

- (i) the fundamental torque with synchronous speed  $N_s$ ,
- (ii) 5th harmonic torque with synchronous speed  $N_s/5$ ,
- (iii) 7th harmonic torque with synchronous speed  $N_s/7$  (provided that higher harmonics are neglected).

Now, 5th harmonic currents will have phase difference of  $5 \times 120 = 600^\circ = 2 \times 360 - 120 = -120^\circ$ . Hence the revolving speed set up will be in reverse direction with speed  $N_s/5$ . The small amount of 5th harmonic torque produces braking action and can be neglected.

The 7th harmonic currents will have phase difference of  $7 \times 120 = 840^\circ = 2 \times 360 + 120 = +120^\circ$ . Hence they will set up rotating field in forward direction with synchronous speed equal to  $N_s/7$ . If we neglect all the higher harmonics, the resultant torque will be equal to sum of fundamental torque and 7th harmonic torque. 7th harmonic torque reaches its maximum positive value just before  $1/7$  th of  $N_s$ . If the mechanical load on the shaft involves constant load torque,



the torque developed by the motor may fall below this load torque. In this case, motor will not accelerate upto its normal speed, but it will run at a speed which is nearly 1/7th of its normal speed. This phenomenon is called as **crawling in induction motors**.

### COGGING

This characteristic of induction motor comes into picture when motor refuses to start at all. Sometimes it happens because of low supply voltage. But the main reason for starting problem in the motor is because of cogging in which the slots of the stator get locked up with the rotor slots.

When the slots of the rotor are equal in number with slots in the stator, they align themselves in such way that both face to each other and at this stage the reluctance of the magnetic path is minimum and motor refuse to start. This **characteristic of the induction motor** is called cogging.

Apart from this, there is one more reason for cogging. If the harmonic frequencies coincide with the slot frequency due to the harmonics present in the supply voltage then it causes torque modulation. As a result, of it cogging occurs. This characteristic is also known as magnetic teeth locking of the induction motor.

### Methods to overcome cogging

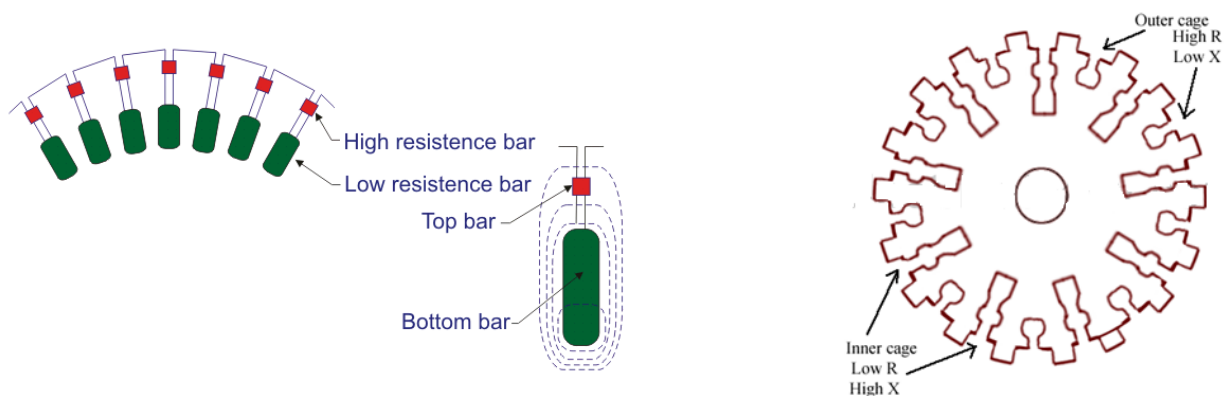
- The number of slots in rotor should not be equal to the number of slots in the stator.
- Skewing of the rotor slots, that means the stack of the rotor is arranged in such a way that it angled with the axis of the rotation.

### 17. Explain with necessary diagrams the principle of operation and characteristics of the double cage induction motor.

Squirrel cage motors are the most commonly used induction motors, but the main drawback in them is their poor starting torque due to low rotor resistance. (Starting torque is directly proportional to the rotor resistance). But increasing the rotor resistance for improving starting torque is not advisory as it will reduce the efficiency of the motor (due to more copper loss). External resistance for starting of purposes cannot be added; as the rotor bars are permanently short circuited. These drawbacks are removed by a **double squirrel cage motor**, which has high starting torque without sacrificing efficiency.

#### Construction Of Double Squirrel Cage Rotor

Rotor of a **double squirrel cage motor** has two independent cages on the same rotor. Bars of high resistance and low reactance are placed in the outer cage, and bars of low resistance and high reactance are placed in the inner cage. The outer cage has high 'reactance to resistance ratio' whereas, the inner cage has low 'reactance to resistance ratio'.



#### Working Of Double Squirrel Cage Motor

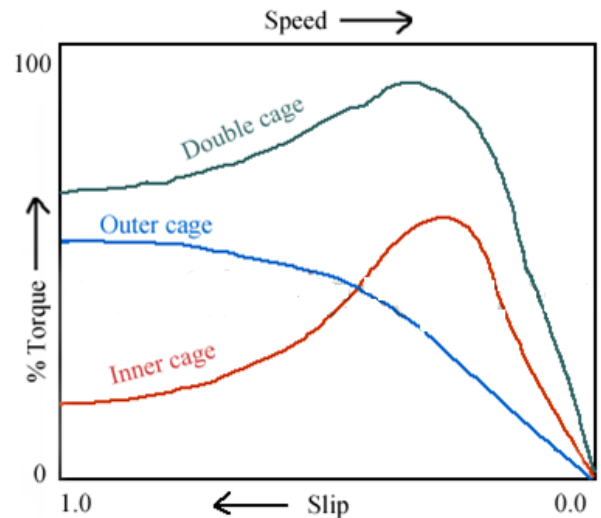
At starting of the motor, frequency of induced emf is high because of large slip (slip = frequency of rotor emf / supply frequency). Hence the reactance of inner cage ( $2\pi fL$  where,  $f$  = frequency of rotor emf) will be very high,



increasing its total impedance. Hence at starting most of the current flows through outer cage despite its large resistance (as total impedance is lower than the inner cage). This will not affect the outer cage because of its low reactance. And because of the large resistance of outer cage starting torque will be large.

As speed of the motor increases, slip decreases, and hence the rotor frequency decreases. In this case, the reactance of inner cage will be low, and most of the current will flow through the inner cage which is having low resistance. Hence giving a good efficiency.

When the double cage motor is running at normal speed, frequency of the rotor emf is so low that the reactance of both cages is negligible. The two cages being connected in parallel, the combined resistance is lower.



The torque speed characteristics of double squirrel cage motor for both the cages are shown in the figure.

**Comparison between single cage and double cage motors:**

1. A double cage rotor has low starting current and high starting torque. Therefore, it is more suitable for direct on line starting.
2. Since effective rotor resistance of double cage motor is higher, there is larger rotor heating at the time of starting as compared to that of single cage rotor.
3. The high resistance of the outer cage increases the resistance of double cage motor. So full load copper losses are increased & efficiency is decreased.
4. The pull out torque of double cage motor is smaller than single cage motor.
5. The cost of double cage motor is about 20-30 % more than that of single cage motor of same rating.

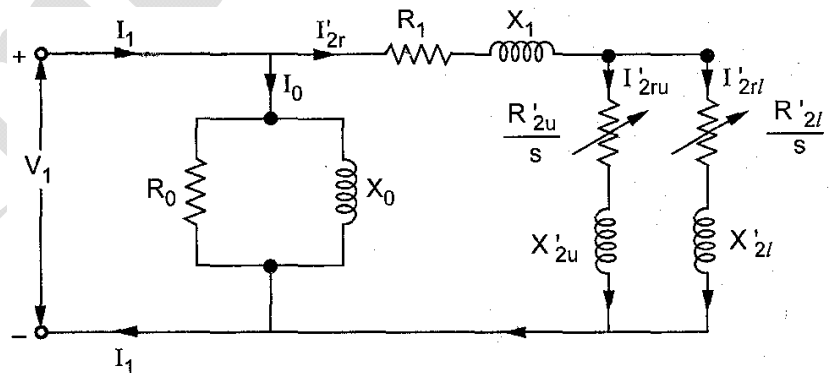
**Equivalent Circuit**

The two cages are assumed to be parallel.

$I'_{2ru}$  and  $I'_{2rl}$  are the currents in the upper and lower cages respectively referred to the stator.

$R'_{2u}$  and  $R'_{2l}$  are the resistances of upper and lower cages respectively referred to the stator.

$X'_{2u}$  and  $X'_{2l}$  are the leakage reactances of upper and lower cages respectively referred to the stator.

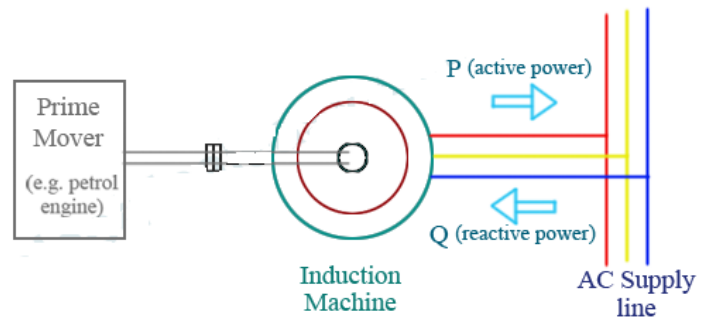


**18. Write a brief note on induction generator.**

If an AC supply is connected to the stator terminals of an induction machine a rotating magnetic field produced in the stator which pulls the rotor to run behind it (the machine is acting as a motor).

If the rotor is accelerated to the synchronous speed by means of a prime mover, the slip will be zero and hence the net torque will be zero. The rotor current will become zero when the rotor is running at synchronous speed.

If the rotor is made to rotate at a speed more than the synchronous speed, the slip becomes negative. A rotor current is generated in the opposite direction, due to the rotor conductors cutting stator magnetic field.



This generated rotor current produces a rotating magnetic field in the rotor which pushes (forces in opposite way) onto the stator field. This causes a stator voltage which pushes current flowing out of the stator winding against the applied voltage. Thus, the machine is now **working as an induction generator (asynchronous generator)**.

Induction generator is not a self-excited machine. Therefore, when running as a generator, the machine takes reactive power from the AC power line and supplies active power back into the line. Reactive power is needed for producing rotating magnetic field. The active power supplied back in the line is proportional to slip above the synchronous speed.

### Self-Excited Induction Generator

An induction machine needs reactive power for excitation, regardless whether it is operating as a generator or a motor. When an induction generator is connected to a grid, it takes reactive power from the grid.

A capacitor bank can be connected across the stator terminals to supply reactive power to the machine as well as to the load. When the rotor is rotated at an enough speed, a small voltage is generated across the stator terminals due to residual magnetism. Due to this small generated voltage, capacitor current is produced which provides further reactive power for magnetization.

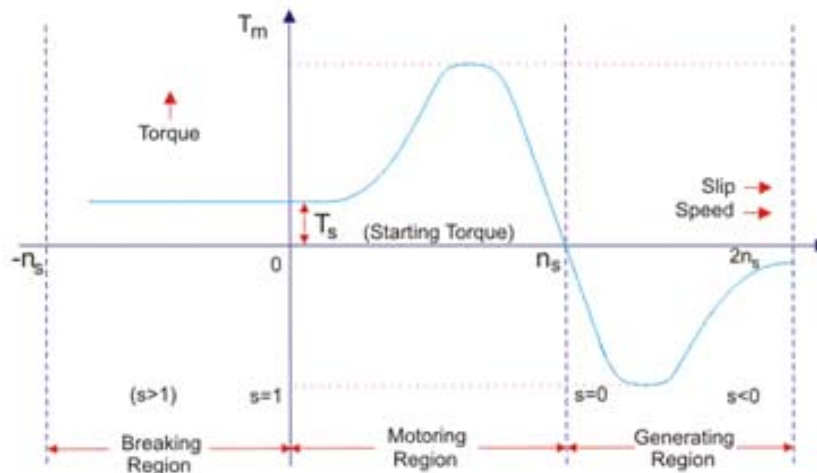
### Applications of induction generators:

Induction generators produce useful power even at varying rotor speeds. Hence, they are suitable in wind turbines.

**Advantages:** Induction or **asynchronous generators** are more rugged and require no commutator and brush arrangement (as it is needed in case of synchronous generators).

**Disadvantage of induction generators** is that they take quite large amount of reactive power.

The torque slip characteristic for motoring and generating action is shown in figure.



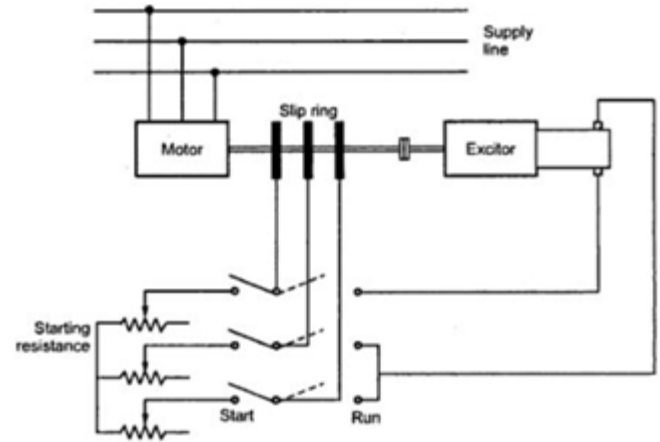
### 19. Write a brief note on synchronous induction motor.

In the applications where high starting torque and constant speed are desired then synchronous induction motors can be used. It has the advantages of both synchronous and induction motors. The synchronous motor gives constant speed whereas induction motors can be started against full load torque.

Consider a normal slip ring induction motor having 3 phase winding on the rotor as shown in fig. The motor is connected to the exciter which gives d.c. supply to the motor through slip rings. One phase carries full d.c. current while the other two carries half of the full d.c. current as they are in parallel. Due to this d.c. excitation, permanent poles (N and S) are formed on the rotor.

Initially it is run as a slip ring induction motor with the help of starting resistances. When the resistance is cut out the motor runs with a slip. Now the connections are changed and the exciter is connected in series with the rotor windings which will remain in the circuit permanently.

As the motor is running as induction motor initially high starting torque (upto twice full load value) can be developed. When d.c. excitation is provided it is pulled into synchronism and starts running at constant speed. The synchronous induction motor provides constant speed, large starting torque, low starting current and power factor correction.

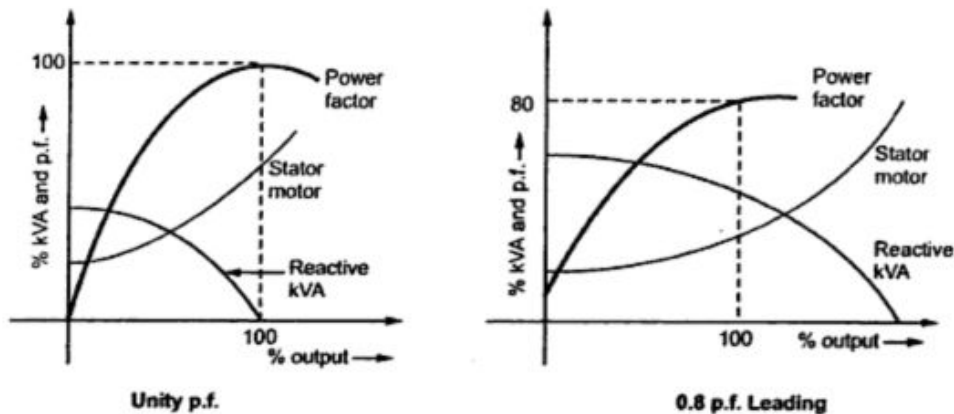


### Performance Characteristics of Synchronous Induction Motors

In the performance characteristics of synchronous induction motor, three different types of torques are to be considered. These are

- the starting torque which indicates capacity of motor to start against load,
- pull in torque which indicates the ability of the motor to maintain operation during change over from induction motor to synchronous motor,
- pull out torque which represents the running of motor synchronously at peak load.

The first two torques are closely related with each other and are the characteristics of the machine running as induction motor. The pull out torque is characteristics when it is running synchronously. The characteristics curves for synchronous induction motor operating at full load unity p.f. and at 0.8 p.f. leading is shown in the Fig.



When the load exceeds the synchronous pull out torque, the machine loses synchronism and runs as an induction motor with fluctuation in torque and slip due to d.c. excitation. With reduction in load torque the motor is automatically resynchronized.

### Advantages of Synchronous Induction Motor

- The synchronous induction motor can start and synchronize against more than full load torque which is not possible with salient pole synchronous motor which must be started against light load.
- The exciter required for synchronous induction motor is of smaller capacity as the gap is not long as compared to normal salient pole motor.
- The rotor winding in synchronous induction motor can function as providing excitation and required damping. So no separate damper winding is required.
- No separate starting and control equipments are required.

### Disadvantages of Synchronous Induction Motor

- i) As the gap is small as compared to normal salient pole synchronous motor it will not give large overload capacity.
- ii) The variation of power factor is large as compared to normal synchronous motor.
- iii) The speed variation is not possible for synchronous induction motor as it runs at constant motor.

### Applications of Synchronous Induction Motor

- The applications where mechanical load is to be driven along with phase advancing properties of synchronous motors are to be used then use of synchronous induction motor is better option.
- applications where in load torque is remaining nearly constant, this motor can be used

### 20. Which tests are required to be performed to obtain the data for the circle diagram? How these tests are performed?

The data required to draw the circle diagram is obtained by conducting (i) no load or open circuit test and (ii) blocked rotor test or short circuit test.

#### No Load Test

In this test, the motor is made to run without any load i.e. no load condition. The speed of the motor is very close to the synchronous speed but less than the synchronous speed. The rated voltage is applied to the stator. The input line current and total input power is measured. The two wattmeter method is used to measure the total input power. The circuit diagram for the test is shown in the Fig. 1.

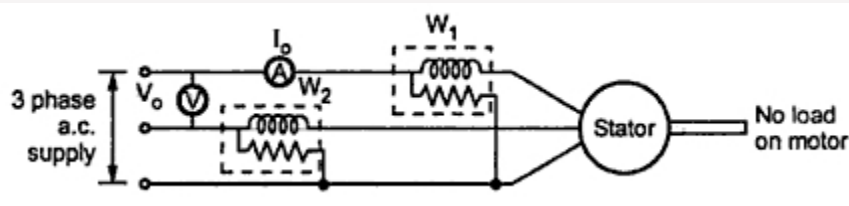


Fig. 1 No load test

The calculations are,

$$W_o = \sqrt{3} V_o I_o \cos \Phi_o$$

$$\text{No load power factor, } \cos \Phi_o = \frac{W_o}{\sqrt{3} V_o I_o}$$

The parameters of the equivalent circuit can be obtained as,

$$I_c = I_o \cos \Phi_o = \text{Active component of no load current}$$

$$I_m = I_o \sin \Phi_o = \text{Magnetising component of no load current}$$

$$R_o = V_o (\text{per phase}) / I_c (\text{per phase}) = \text{No load branch resistance}$$

$$X_o = V_o (\text{per phase}) / I_m (\text{per phase}) = \text{No load branch resistance}$$

The power input  $W_o$  consists of following losses,

1. Stator copper loss i.e.  $3 I_o^2 R_1$  where  $I_o$  is no load per phase current and  $R_1$  is stator resistance per phase.
2. Stator core loss i.e. iron loss.
3. Friction and windage loss.

Under no load condition,  $I_o$  is also very small and in many practical cases it is also neglected. Thus  $W_o$  consists of stator iron loss and friction and windage loss which are consists for all load conditions. Hence  $W_o$  is said to give fixed losses of the motor.

$$\therefore W_o = \text{No load power input} = \text{Fixed loss}$$

#### Blocked Rotor Test

In this test, the rotor is locked and it is not allowed to rotate. Thus the slip  $s = 1$  and  $R_L' = R_2' (1-s)/s$  is zero. If the motor is slip ring induction motor then the windings are short circuited at the slip rings.

A reduced voltage (about 10 to 15 % of rated voltage) is applied such that stator carries rated current. Now the applied voltage  $V_{sc}$ , the input power  $W_{sc}$  and a short circuit current  $I_{sc}$  are measured.

As  $R_L' = 0$ , the equivalent circuit is exactly similar to that of a transformer and hence the calculations are similar to that of short circuit test on a transformer.

$V_{sc}$  = Short circuit reduced voltage (line value)

$I_{sc}$  = Short circuit current (line value)

$W_{sc}$  = Short circuit input power

Now  $W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \Phi_{sc}$  .....Line values

Short circuit power factor of a motor  $\cos \Phi_{sc} = \frac{W_{sc}}{\sqrt{3} V_{sc} I_{sc}}$

$$W_{sc} = 3 (I_{sc})^2 R_{1e}$$

where  $I_{sc}$  = Per phase value

Equivalent resistance referred to stator  $R_{1e} = \frac{W_{sc}}{3(I_{sc})^2}$

Equivalent impedance referred to stator,  $Z_{1e} = V_{sc} \text{ (per phase)} / I_{sc} \text{ (per phase)}$

Equivalent reactance referred to stator,  $X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$

During this test, the stator carries rated current hence the stator copper loss is also dominant. Similarly the rotor also carries short circuit current to produce dominant rotor copper loss. As the voltage is reduced, the iron loss which is proportional to voltage is negligibly small. The motor is at standstill hence mechanical loss i.e. friction and windage loss is absent. Hence we can write,

$$W_{sc} = \text{Stator copper loss} + \text{Rotor copper loss}$$

But it is necessary to obtain short circuit current when normal voltage is applied to the motor. This is practically not possible. But the reduced voltage test results can be used to find current  $I_{SN}$  which is short circuit current if normal voltage is applied.

If  $V_L$  = Normal rated voltage (line value)

$V_{sc}$  = Reduced short circuit voltage (line voltage)

$$\text{Then } I_{SN} = (V_L / V_{sc}) \times I_{sc}$$

where  $I_{sc}$  = Short circuit current at reduced voltage

Thus,  $I_{SN}$  = Short circuit current at normal voltage

Now power input is proportional to square of the current.

So  $W_{SN}$  = Short circuit input power at normal voltage

This can be obtained as,

$$W_{SN} = (I_{SN} / I_{sc})^2 \times W_{sc}$$

But at normal voltage core loss can not be negligible hence,

$$W_{SN} = \text{Core loss} + \text{Stator and rotor copper loss}$$

## 21. Explain the procedure of drawing circle diagram of an induction motor. What information can be drawn from the circle diagram and how?

By using the data obtained from the no load test and the blocked rotor test, the circle diagram can be drawn using the following steps:

**Step 1:** Take reference phasor  $V$  as vertical (Y-axis).

**Step 2:** Select suitable current scale such that diameter of circle is about 20 to 30 cm.

**Step 3:** From no load test,  $I_o$  and  $\Phi_o$  are obtained. Draw vector  $I_o$ , lagging  $V$  by angle  $\Phi_o$ . This is the line  $OO'$  as shown in the Fig.

**Step 4:** Draw horizontal line through extremity of  $I_o$  i.e.  $O'$ , parallel to horizontal axis.

**Step 5:** Draw the current  $I_{SN}$  calculated from  $I_{sc}$  with the same scale, lagging  $V$  by angle  $\Phi_{sc}$ , from the origin  $O$ . This is phasor  $OA$  as shown in the Fig.

**Step 6:** Join  $O'A$ . the line  $O'A$  is called output line.

**Step 7:** Draw a perpendicular bisector of O'A. Extend it to meet line O'B at point C. This is the centre of the circle.

**Step 8:** Draw the circle, with C as a center and radius equal to O'C. This meets the horizontal line drawn from O' at B as shown in the Fig.

**Step 9:** Draw the perpendicular from point A on the horizontal axis, to meet O'B line at F and meet horizontal axis at D.

**Step 10:** Torque line.

The torque line separates stator and rotor copper losses.

Thus the vertical distance AD represents power input at short circuit i.e.  $W_{SN}$ , which consists of core loss and stator, rotor copper losses.

Now  $FD = O'G =$  Fixed loss

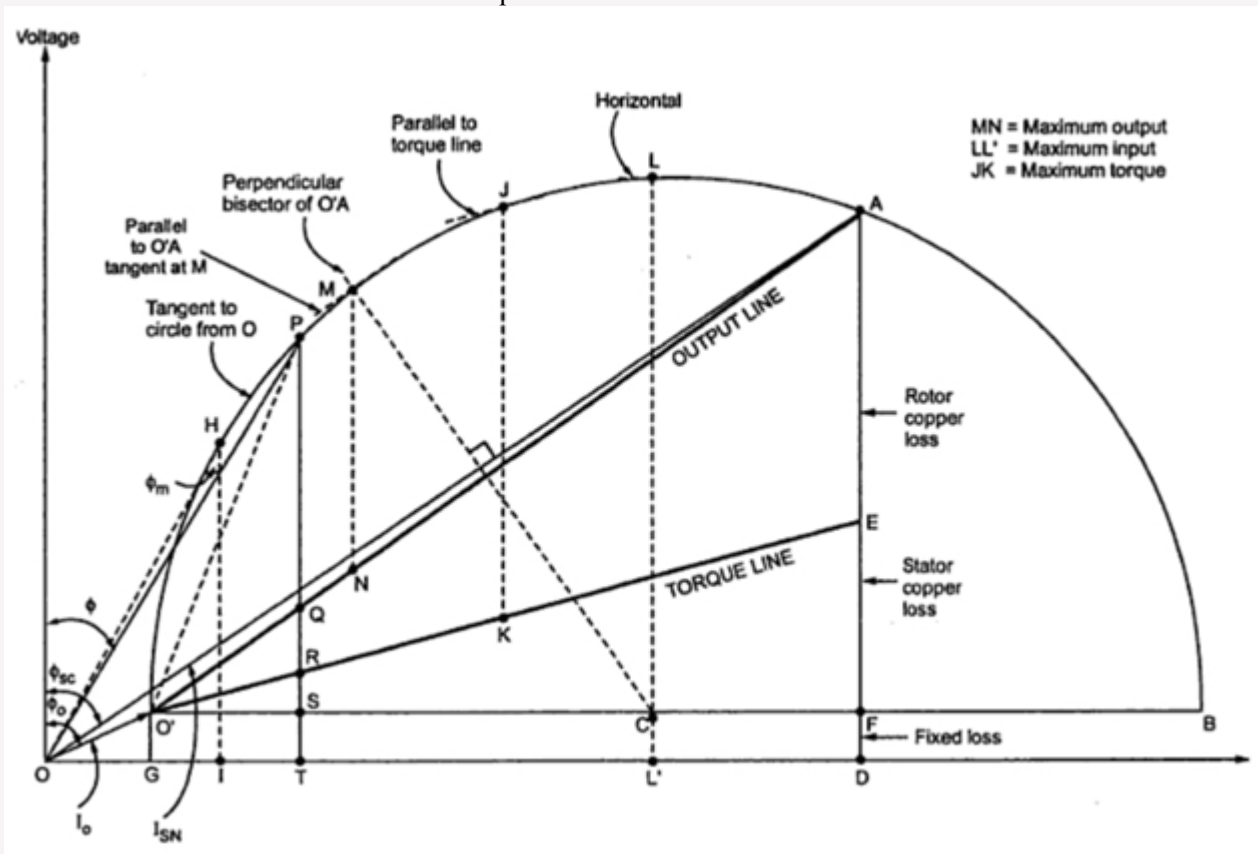
Where O'G is drawn perpendicular from O' on horizontal axis. This represents power input on no load i.e. fixed loss.

Hence  $AF \propto$  Sum of stator and rotor copper losses

Then point E can be located as,

$$\frac{AE}{EF} = \frac{\text{Rotor copper loss}}{\text{Stator copper loss}}$$

The line O'E under this condition is called torque line.



**Power scale:** As AD represents  $W_{SN}$  i.e. power input on short circuit at normal voltage, the power scale can be obtained as,

$$\text{Power scale} = \frac{W_{SN}}{l(AD)} \text{ W/cm}$$

where  $l(AD) =$  Distance AD in cm

**Location of Point E:** In a slip ring induction motor, the stator resistance per phase  $R_1$  and rotor resistance per phase  $R_2$  can be easily measured. Similarly by introducing ammeters in stator and rotor circuit, the currents  $I_1$  and  $I_2$  also can be measured.

∴  $K = I_1/I_2 = \text{Transformation ratio}$

Now  $\frac{AE}{EF} = \frac{\text{Rotor copper loss}}{\text{Stator copper loss}} = \frac{I_2^2 R_2}{I_1^2 R_1} = \frac{R_2}{R_1} \left[ \frac{I_2}{I_1} \right]^2 = \frac{R_2}{R_1} \cdot \frac{1}{K^2}$

But  $R_2' = R_2/K^2 = \text{Rotor resistance referred to stator}$

∴  $AE/EF = R_2'/R_1$

Thus point E can be obtained by dividing line AF in the ratio  $R_2'$  to  $R_1$ .

In a **squirrel cage motor**, the stator resistance can be measured by conducting resistance test.

∴ Stator copper loss =  $3I_{SN}^2 R_1$  where  $I_{SN}$  is phase value.  
 Neglecting core loss,  $W_{SN} = \text{Stator Cu loss} + \text{Rotor Cu loss}$

∴ Rotor copper loss =  $W_{SN} - 3I_{SN}^2 R_1$

∴  $AE/EF = (W_{SN} - 3I_{SN}^2 R_1) / (3I_{SN}^2 R_1)$

Dividing line AF in this ratio, the point E can be obtained and hence O'E represents torque line.

**Predicting Performance Form Circle Diagram**

Let motor is running by taking a current OP as shown in the Fig. The various performance parameters can be obtained from the circle diagram at that load condition.

Draw perpendicular from point P to meet output line at Q, torque line at R, the base line at S and horizontal axis at T.

Using the power scale and various distances, the values of the performance parameters can be obtained as,

- Total motor input = PT x Power scale
- Fixed loss = ST x power scale
- Stator copper loss = SR x power scale
- Rotor copper loss = QR x power scale
- Total loss = QT x power scale
- Rotor output = PQ x power scale
- Rotor input = PQ + QR = PR x power scale
- Slip  $s = \text{Rotor Cu loss} = QR/PR$
- Power factor  $\cos = PT/OP$
- Motor efficiency = Output / Input = PQ/PT
- Rotor efficiency = Rotor output / Rotor input = PQ/PR
- Rotor output / Rotor input =  $1 - s = N/N_s = PQ/PR$
- The torque is the rotor input in synchronous watts.

**Maximum Quantities**

The maximum values of various parameters can also be obtained by using circle diagram.

**1. Maximum Output:** Draw a line parallel to O'A and is also tangent to the circle at point M. The point M can also be obtained by extending the perpendicular drawn from C on O'A to meet the circle at M. Then the maximum output is given by l(MN) at the power scale. This is shown in the Fig.

**2. Maximum Input:** It occurs at the highest point on the circle i.e. at point L. At this point, tangent to the circle is horizontal. The maximum input given l(LL') at the power scale.

**3. Maximum Torque:** Draw a line parallel to the torque line and is also tangent to the circle at point J. The point J can also be obtained by drawing perpendicular from C on torque line and extending it to meet circle at point J. The l(JK) represents maximum torque in synchronous watts at the power scale. This torque is also called stalling torque or pull out torque.

**4. Maximum Power Factor:** Draw a line tangent to the circle from the origin O, meeting circle at point H. Draw a perpendicular from H on horizontal axis till it meets it at point I. Then angle OHI gives angle corresponding to maximum power factor angle.

∴ Maximum p.f. =  $\cos \{ \angle OHI \} = HI/OH$

**5. Starting Torque:** The torque is proportional to the rotor input. At  $s = 1$ , rotor input is equal to rotor copper loss i.e. l(AE).

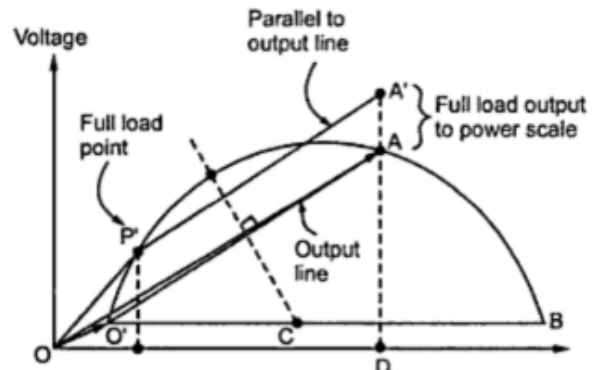
∴  $T_{start} = l(AE) \times \text{Power scale}$  .....in synchronous watts

### Full load Condition

The full load motor output is given on the name plates in watts or h.p. Calculate the distance corresponding to the full load output using the power scale.

Then extend AD upwards from A onwards, equal to the distance corresponding to full load output, say A'. Draw parallel to the output line O'A from A' to meet the circle at point P'. This is the point corresponding to the full load condition, as shown in the Fig.

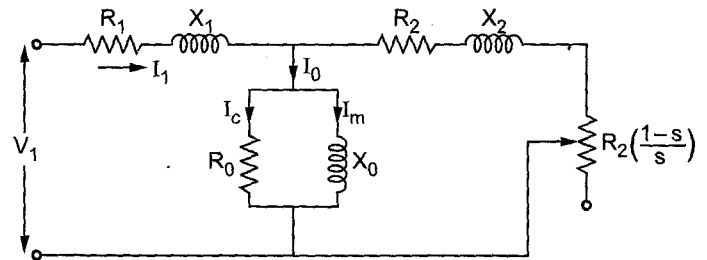
Once point P' is known, the other performance parameters can be obtained easily as discussed above.



## 22. Obtain equivalent circuit parameters from the no load and blocked rotor test on induction motor.

### NO LOAD TEST

- No load power factor,  $\cos \phi_0 = W_0 / V_0 I_0$   
 $V_0$  – No load voltage per phase in volts  
 $I_0$  – No load current per phase in amps  
 $W_0$  – No load power per phase in watts
- Active Component,  $I_c = I_{0(ph)} \cos \phi_0$
- Magnetising Component,  $I_m = I_{0(ph)} \sin \phi_0$
- No load resistance ( $R_0$ ) =  $V_0 / I_c \ \Omega$
- No load reactance ( $X_0$ ) =  $V_0 / I_m \ \Omega$



### BLOCKED ROTOR TEST

- Equivalent impedance referred to stator,  $Z_{1e(ph)} = V_{SC(ph)} / I_{SC(ph)} \ \Omega$
- Equivalent resistance referred to stator,  $R_{1e(ph)} = W_{SC(ph)} / [3 I_{SC(ph)}^2] \ \Omega$
- Equivalent reactance referred to stator,  $X_{1e(ph)} = \sqrt{[(Z_{1e(ph)})^2 - (R_{1e(ph)})^2]} \ \Omega$
- Rotor resistance referred to stator ( $R'_{2(ph)} = R_{1e(ph)} - R_1 \ \Omega$ )
- Rotor reactance referred to stator ( $X'_{2(ph)} = X_{1e(ph)} / 2 = X_1 \ \Omega$ )

Where  $R_1$  - stator resistance per phase

$X_1$  – stator reactance per phase

$R_1 = R_{(ac)} = 1.6 R_{(dc)}$

- Equivalent load resistance,  $R'_L = R'_2 (1/s - 1) \ \Omega$

Where slip,  $s = (N_s - N) / N_s$

$N_s$  – Synchronous speed in rpm

$N$  – Speed of motor in rpm